



A computational fluid dynamics study on the heat transfer characteristics of the working cycle of a low-temperature-differential γ -type Stirling engine



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ABSTRACT

A three-dimensional compressible CFD code has been developed to study the heat transfer characteristics of a twin-power piston γ -type Stirling engine. The results include temperature contours, velocity vectors, and distributions of local heat flux along solid boundaries at several important time steps as well as variation of average temperatures, integrated rates of heat input, heat output, and engine power. It is found that Impingement is the major heat transfer mechanism in the expansion and compression chambers, and the temperature distribution is highly non-uniform across the engine volume at any given moment. This study sheds light into the complex heat transfer mechanism inside the Stirling engine and is very helpful to the understanding of the fundamental process of the engine cycle.

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1. Introduction

Stirling engine, a nearly 200 years old invention, has attracted much attention and been the subject of intensive research by many industrial and academic institutes in recent decades due to its many specific advantages [1–4]. These advantages are: very high efficiency, silence in operation, low level of toxic emission if powered by fuel, and the ability to use almost any kind of heat sources, including solar, biological, geothermal, or even industrial waste heat. Especially the last advantage makes it regarded as an important green energy device and potentially one of the most promising solutions to the global warming problem. However, there are also disadvantages associated with the Stirling engine. It is low in power/weight ratio, unable to change torque and power swiftly, and expensive (especially for those associated with concentration solar energy); and these have preclude the engine's widespread into some of today's important applications such as transportation and electricity generation. Despite these disadvantages, Stirling engine technology has been constantly improved to produce new generation of engines with higher power/weight ratio and higher power output and efficiency; and it might become more popular and widespread into more applications in the future.

Improvement on Stirling engine's performance relies heavily on extensive numerical and experimental research on engine cycle. To

design a new Stirling engine, a numerical analysis often precedes experiment to find some general directions to follow. It is also a more suitable tool than experiment to carry out parameter optimization. Therefore, many numerical models for Stirling engine cycle have been developed. A detailed overview of the existing numerical models for the analysis on different Stirling engine cycles can be found in Mahkamov [5]. These models are basically classified as the first-, second-, and third-order models, according to the model hierarchy. Here, the term "order" has nothing to do with the mathematical terms in the governing equations. In these models, the entire engine is divided into 3, or 5, or more sections (control volumes), so they are practically zero- or one-dimensional models. Among them, the model proposed by Schmidt is the stereotype, and improved variants of Schmidt's model are nowadays the most widely adopted models [6–8] for numerical analysis of Stirling engines. Some second- or third-order models take non-isothermal and transient effects into account and are capable of predicting the average values of volume, pressure, and heat transfer rate in each section at each time step of the cycle, allowing the overall heat input and power output in a cycle to be calculated. However, because these models are zero- or one-dimensional models, they, at best, only resolve spatial variations in the axial direction of the engine. In this case, any variation in the transverse direction that is important for predicting the engine's performance accurately will not be accounted for. Neither can they resolve the effects caused by changes in some geometrical features such as the shapes and dimensions of displacer and displacer cylinder,

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Nomenclature

c_p	constant pressure specific heat ($\text{kJ kg}^{-1} \text{K}^{-1}$)	t	time (s)
k	thermal conductivity ($\text{Wm}^{-1} \text{K}^{-1}$)	t_p	time period of an engine cycle
l_1	length of the power piston linkage bar (m)	T	temperature (K)
l_2	length of the power piston connection rod (m)	u, v, w	velocity components respectively in x-, y-, and z-directions (ms^{-1})
l_3	length of the displacer linkage bar (m)	$\tilde{u}, \tilde{v}, \tilde{w}$	local frame moving velocity components respectively in x-, y-, and z-directions (ms^{-1})
l_4	length of the displacer connection rod (m)	V	volume (m^3)
l_d	height of displacer (m)	W	engine power (W)
m	mass (kg)	x, y, z	components of Cartesian coordinate system (m)
p	pressure (Pa)		
Q	heat transfer rate (W)	Greeks	
q	heat flux or heat transfer rate per unit area (Wm^{-2})	β	crank angle of the displacer
R	gas constant ($\text{Jkg}^{-1} \text{K}^{-1}$)	ρ	density (kg m^{-3})
R_1	outer radius of the power piston (m)	η	engine efficiency
R_2	inner radius of the displacer cylinder (m)	θ	crank angle
R_d	outer radius of the displacer (m)	μ	viscosity (Pas)
R_λ	thermal conductivity ratio between solid and gas materials	ω	engine speed (rads^{-1})
r_1	crank radius of the power piston (m)		
r_2	crank radius of the displacer (m)		

regeneration chamber, or the internal gas circuits, all of which can only be defined in multi dimensions. This means that they are not very accurate in terms of the optimization of the geometries and dimensions of individual components during the design of a Stirling engine. In addition, heat transfer rates inside the expansion and compression chambers are often estimated by using constant convective heat transfer coefficients, a practice that has oversimplified the complexity of the real heat transfer mechanism taking place inside these chambers. These factors together with other simplifications result in poor predictive accuracy returned by these models. They tend to over-predict engine's output power and efficiency by the margins from 30% to 100%, or even more.

A Stirling cycle involves very complicated heat and mass transfer processes, and the engine itself consists of many multi-dimensional components whose geometrical effects can be important. None of these can be resolved in great details or very accurately by those aforementioned numerical models. Yet, resolving these sophisticated physical processes and the effects from some geometrical features are crucial to the prediction of the overall engine performance. Especially understanding the effects of the geometrical features is of practical importance because to build a new Stirling engine, engineers have to determine the detail geometries and dimensions of all components in the engine. Therefore, more elaborated models are needed to improve the accuracy of numerical analysis. Computational fluid dynamics (CFD) approach is one of such numerical models by far that are capable of simulating those complicated processes taking place in Stirling engine and hence returning more accurate prediction on the overall engine performance. It is also capable of resolving the effects caused by changes in small geometrical features and potentially be a great tool for the design and development process of a new Stirling engine. However, there exist many challenges associated with applying a CFD approach on a Stirling cycle. Although a Stirling engine is a close system where precise boundaries can be defined, the problem is unsteady, multiphase, and multi-scale in nature. Furthermore, the engine involves moving parts such as displacer and power piston; hence the volume of computational domain is changing with time. These bring challenges in two aspects. In the aspect of coding, simulating the reciprocating motions of displacer and power piston requires the use of moving mesh techniques which introduce complex algorithm and raise the difficulty of coding. In the aspect flow physics, the gas flow is compressible; the heat transfer mechanism

is conjugate heat transfer, involving solid and fluid media; and in most cases, the flow is turbulent; and these give rise to complex heat and mass transfer processes. To resolve these processes, it is necessary to use very fine grids and small time steps and solve many coupled governing equations at the same time, resulting in problems of convergence and long CPU hours. Due to these challenges, many attempts to apply CFD approach on Stirling engines focused on separated components such as combustion chamber of the heater [9]. There have been very limited reports on using CFD approach to study the full cycle of a Stirling engine. Mahkamov [5] used both second-order approach and axisymmetric CFD approach to analyze the working process of a solar Stirling engine. They found significant differences in temperature data and engine performance obtained by both methods. The transient temperature variations by CFD approach are far from the harmonic distributions given by the second-order approach, and CFD model returns much more accurate output power than the second-order model. Mahkamov [10] also reported a 3D CFD approach on a prototype biomass Stirling engine to improvement its performance. The engine was also analyzed by the first-order and second-order approaches. The CFD method identified some key factors that reduce the power of the engine: a geometrical feature that causes high level of hydraulic losses in the regenerator, and the entrapment of the gas in the pipe connecting two parts of the compression space and the dead volume introduced by the pipe. Such detailed analysis on the effects imposed by multi-dimensional geometrical features can only be resolved by a CFD approach. The study also highlighted the usefulness of adopting a CFD approach to identify and solve problems during the design of a Stirling engine.

Among different types of Stirling engines, the γ -type engine has many specific advantages as described in Kongtragool and Wongwises [11]. It is a low to medium temperature differential engine; therefore it can run on temperature difference from just tens of degree to a couple of hundreds of degree. This is a very important advantage because the engine can be powered by a wide range of energy sources, including low-grade sources such as solar, bio-fuel, geothermal, or even industrial waste heat. It is also much easier and cheaper to be constructed and maintained than other types of Stirling engines. Therefore, the γ -type engine is very suitable for domestic use, which can potentially become a very large market, because of the readily availability and low cost of low-grade fuel and the engine's simplicity to construct and maintain. Although

2. Mathematical model

where $\beta = \theta - 90^\circ$. The velocities of piston and displacer are:

$$w_p(\theta) = r_1 \omega \cos \theta - \frac{r_1^2 \omega \cos \theta \sin \theta}{\sqrt{l_1^2 - r_1^2 \cos^2 \theta}}, \quad (3)$$

$$w_d(\beta) = r_2 \omega \cos \beta - \frac{r_2^2 \omega \cos \beta \sin \beta}{\sqrt{l_3^2 - r_2^2 \cos^2 \beta}}. \quad (4)$$

The configuration of the computational domain, expressed by the computational mesh, is illustrated in Fig. 2. As seen, due to geometrical symmetry, only a quarter of the real engine space is required. Since the engine in Chen's [13] experiment is quite large, the flow inside the engine could become turbulent at relatively low engine speed. Therefore in this study, the engine has been scaled down by a half to reduce Reynolds number and retain laminar flow inside the engine, precluding the need to involve turbulence modeling. In addition, due to its smaller size, the engine is not equipped with a regenerator, and regeneration is performed by the working gas flowing forwards and backwards between the expansion and compression chambers through the narrow gap formed by the displacer and the inner wall of displacer cylinder; that is, the solid material of the wall of displacer cylinder is acting like regenerator material to store and release heat. Furthermore, due to the fact that the heat capacity of solid wall is much larger than that of the gas, the temperature of solid wall is further assumed to retain at a constant value (on the hot- or cold-end wall) or a fixed profile along the z-direction (on the displacer cylinder wall), hence eliminating the need to include the solid domain. Some other assumptions to facilitate CFD simulation are made as follows: fluid viscosity and thermal properties all materials are assumed constant, the working gas is air and assumed to be ideal gas, and effects from mechanical friction and thermal radiation are ignored. Therefore, the flow and heat transfer phenomena of the engine cycle can be solved by transient three-dimensional laminar and compressible Navier–Stokes equations plus energy equation and ideal gas equation of state as follows:

Continuity equation :

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho \tilde{u}) + \frac{\partial}{\partial x}(\rho \tilde{v}) + \frac{\partial}{\partial x}(\rho \tilde{w}) = 0. \quad (5)$$

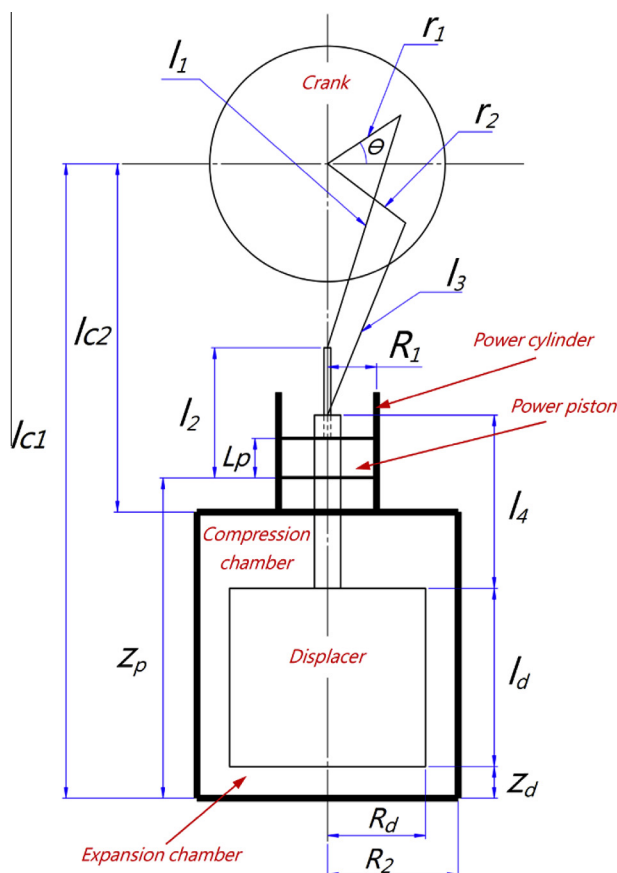


Fig. 1. The configuration and definition of geometric parameters of the γ -type Stirling engine.

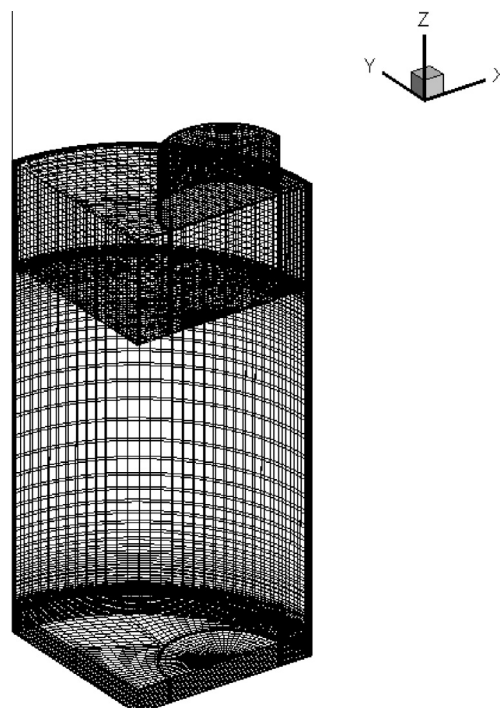


Fig. 2. The computational mesh, showing the configuration of the computational domain.

Momentum equations:

$$\begin{aligned} \frac{\partial(\rho u)}{\partial t} + \frac{\partial}{\partial x}(\rho \tilde{u}u) + \frac{\partial}{\partial x}(\rho \tilde{v}u) + \frac{\partial}{\partial x}(\rho \tilde{w}u) \\ = -\frac{\partial p}{\partial x} + \mu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right), \end{aligned} \quad (6)$$

$$\begin{aligned} \frac{\partial(\rho v)}{\partial t} + \frac{\partial}{\partial x}(\rho \tilde{u}v) + \frac{\partial}{\partial x}(\rho \tilde{v}v) + \frac{\partial}{\partial x}(\rho \tilde{w}v) \\ = -\frac{\partial p}{\partial y} + \mu \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right), \end{aligned} \quad (7)$$

$$\begin{aligned} \frac{\partial(\rho w)}{\partial t} + \frac{\partial}{\partial x}(\rho \tilde{u}w) + \frac{\partial}{\partial x}(\rho \tilde{v}w) + \frac{\partial}{\partial x}(\rho \tilde{w}w) \\ = -\frac{\partial p}{\partial z} - \rho g + \mu \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right). \end{aligned} \quad (8)$$

Energy equation:

$$\begin{aligned} \frac{\partial(\rho T)}{\partial t} + \frac{\partial}{\partial x}(\rho \tilde{u}T) + \frac{\partial}{\partial x}(\rho \tilde{v}T) + \frac{\partial}{\partial x}(\rho \tilde{w}T) \\ = \frac{k}{c_p} \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right). \end{aligned} \quad (9)$$

Equation of state:

$$p = \rho RT. \quad (10)$$

In the above, $\tilde{u} = u - u_b$, $\tilde{v} = v - v_b$, $\tilde{w} = w - w_b$ are relative velocity components between fluid and local moving frame that moves with u_b , v_b , w_b , respectively in x , y , and z directions. In Eq. (9), the viscous dissipation effect has not been included. As argued in Mahkamov [10], this effect is insignificant compared with other heat transfer effects in Stirling engine, thus it can be neglected. The initial conditions are:

$$\theta = 0^\circ, \quad u = v = w = 0, \quad p = 101.0 \text{ kPa}, \quad T = T_L. \quad (11)$$

Boundary conditions are:

At hot end, $z = 0$:

$$u = v = w = 0, \quad T = T_H. \quad (12)$$

At cold end, $z = l_{c1} - l_{c2}$ and on the wall of power cylinder:

$$u = v = w = 0, \quad T = T_L. \quad (13)$$

The surface of displacer is assumed adiabatic, thus the conditions on displacer surfaces are:

$$u = v = 0, \quad w = w_d, \quad \frac{\partial T}{\partial n} = 0, \quad (14)$$

where n is the direction normal to the wall of displacer. The surface of power piston is also assumed adiabatic, giving:

$$u = v = 0, \quad w = w_p, \quad \frac{\partial T}{\partial z} = 0. \quad (15)$$

The temperature on the lateral wall of the displacer cylinder (the wall of regenerative channel) is assumed to maintain at a fixed linear profile as:

$$u = v = w = 0, \quad T = T_L + \frac{z}{(l_{c1} - l_{c2})} (T_H - T_L). \quad (16)$$

3. Numerical procedure

The numerical procedure in this study is based on an in-house unstructured-mesh, fully collocated, finite-volume code, 'USTREAM' developed by the corresponding author. This is the descendent of the structured-mesh, multi-block code of 'STREAM'

[14]. In a Stirling engine, the actions of the displacer and the power pistons cause expansion and compression of the engine volume, and a moving mesh technique is needed to simulate such variations in volume. There are two commonly used moving mesh techniques: one is adding or deleting cells to change the volume of the computational domain, and the other is to directly change the geometries of cells to expand or compress their volumes. In this study, the second strategy is adopted. The number of cells in the computational domain is fixed, and the expansion or compression of engine volume is performed by moving the local boundaries of cells to increase or decrease the volumes of the cells, and collectively, they can simulate the expansion or compression of the engine volume just like the action of an accordion. The moving of cell boundary creates local moving frame velocity components $\tilde{u}$, $\tilde{v}$, $\tilde{w}$. However, only the component $\tilde{w}$ exists due to the fact that the motions of displacer and power piston are both along the z -direction.

Another important issue is to simulate the compressible flow inside the engine. Since USTREAM is a pressure-based SIMPLEC [15] finite volume code, the pressure correction equation has to be modified to account for variable density in compressible flows. The pressure-velocity coupling scheme in Lebon et al. [16] is adopted here to perform this task.

4. Results and discussion

The dimensions of the simulated γ -type Stirling engine are given in Table 1. The working gas is air with a gas constant of

Table 1
The values of the geometrical parameters of the Stirling engine.

R_1 (m)	0.0128
R_2 (m)	0.0400
R_d (m)	0.0390
r_1 (m)	0.0200
r_2 (m)	0.0125
l_1 (m)	0.0650
l_2 (m)	0.0225
l_3 (m)	0.0475
l_4 (m)	0.0775
l_d (m)	0.0740
l_{c1} (m)	0.2125
l_{c2} (m)	0.1105

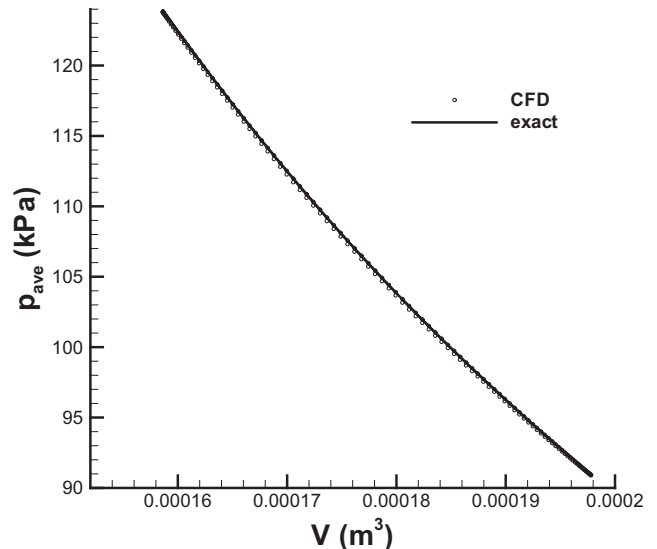


Fig. 3. p_{ave} - V diagram of an adiabatic cycle of the γ -type Stirling engine for code validation purpose.

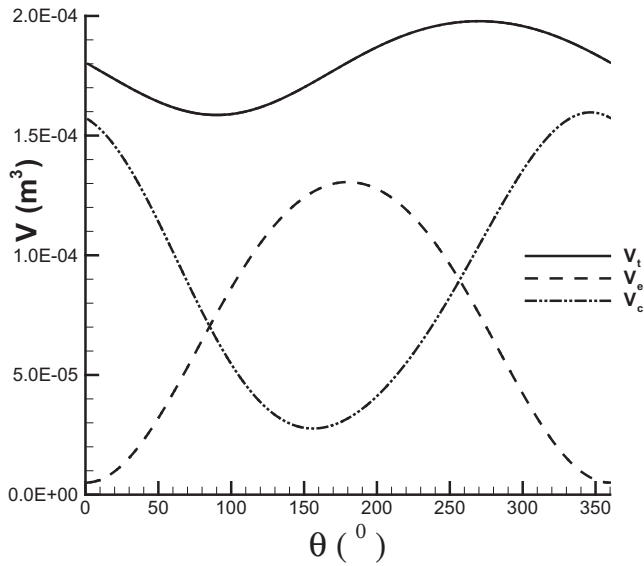


Fig. 4. Variations of total, expansion chamber, compression chamber volumes versus crank angle within an engine cycle.

287.0 Jkg⁻¹ K⁻¹, and the initial crank angle and gas pressure are 0.0 degree and 101.0 kPa, respectively. The operation conditions are: the rotation speed of the engine is -120.0 rpm (clockwise); the cold-end and hot-end temperatures are 300.0 K and 400.0 K, respectively. The first step of any CFD simulation is to determine a proper grid and time step interval to obtain grid and time-step independent solutions. Three different grids with cell numbers of 80,566, 110,569, and 141,438, respectively were tested by using three different time-step intervals of 2.5×10^{-3} s, 1.667×10^{-3} s, and 1.25×10^{-3} s, which respectively correspond to time-step numbers per cycle of 200, 300, and 400. Fig. 2 shows the grid with 80,566 cells. As seen, grid density has been concentrated towards all solid boundaries to resolve the flow and thermal boundary layers. For each simulation, about 4 cycles were needed for the solution to become periodical. Judging from the values of maximum velocity, heat input, and work output within a cycle, grid and time-step independent solutions can be achieved by the grid with 110,569 cells at 200 time steps per cycle. The three quantities obtained by this grid and time-step interval are less than 0.5% in difference with those obtained with the finest grid and smallest time-step interval; therefore this grid and the time-step interval have been used in this study. To validate the correctness of

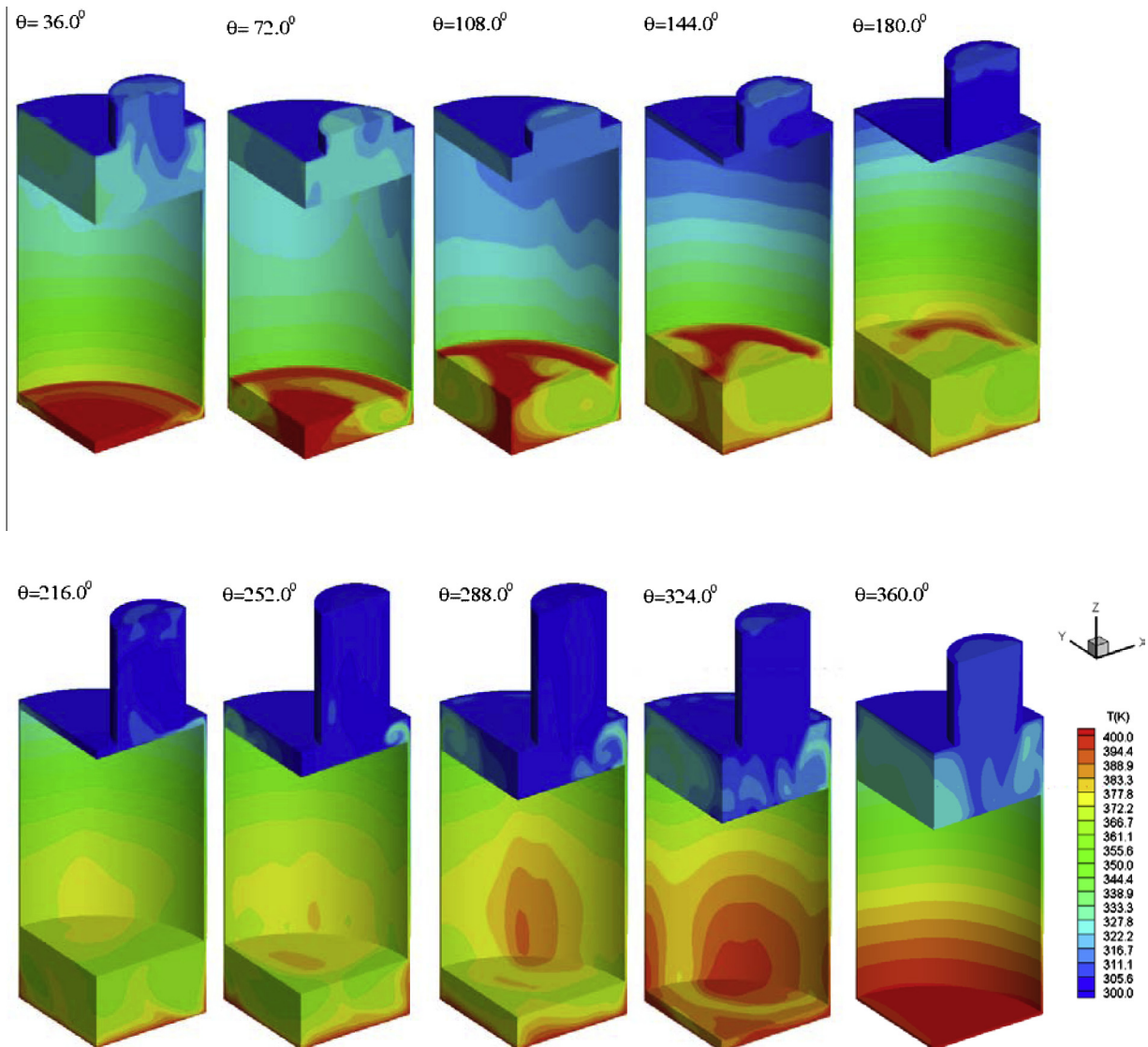


Fig. 5. Temperature contours across the whole engine at 10 different crank angles.

USTREAM code, an adiabatic cycle of the current engine has been performed, and the $p_{ave}-V$ diagram is shown in Fig. 3. For an adiabatic cycle, the exact relation between p and V is $pV^{1.4} = C$, where C is a constant. Fig. 3 indicates that USTREAM code returns results that fit this relation very well; hence, the correctness of the code has been validated.

Fig. 4 shows the variations of total, expansion chamber, and compression chamber volumes versus crank angle over an engine cycle. The maximum and minimum values of the total volume are respectively $1.586 \times 10^{-4} \text{ m}^3$ and $1.979 \times 10^{-4} \text{ m}^3$, resulting in a compression ratio of 1.248, and they occur at $\theta = 90.0^\circ$ and 270.0° , respectively. Since the expansion chamber includes the power cylinders, the maximum volume of the compression chamber is larger than that of the expansion chamber.

To demonstrate the general variations of temperature within the engine over an engine cycle, Fig. 5 shows the temperature contours of the entire engine at 10 different crank angles, from $\theta = 36.0^\circ$ to 360.0° with an increment of 36.0° . The motions of

displacer and power pistons pose different effects on the flow inside the engine. The combined motions of displacer and power pistons cause gas to flow forwards and backwards between the expansion and compression chambers, while the motion of the power pistons expands or compresses the total volume of the engine, which in turn introduces cooling or heating effects on the working gas. The gas that flows forwards and backwards between expansion and compression chambers introduces very strong heat convection effect in both chambers and results in complicated heat transfer behaviors between the gas and the solid boundaries and highly non-uniform temperature distributions across the chambers at any given moment. In addition, due to the fact displacer and power cylinders are eccentric, the temperature distributions are not axisymmetric, and there exist pronounced 3D phenomena throughout the entire engine cycle. To this end, it is readily realized that the assumption of uniform temperature distribution and the adoption of constant convective heat transfer coefficients to account for the heat transfer effect in expansion and compression

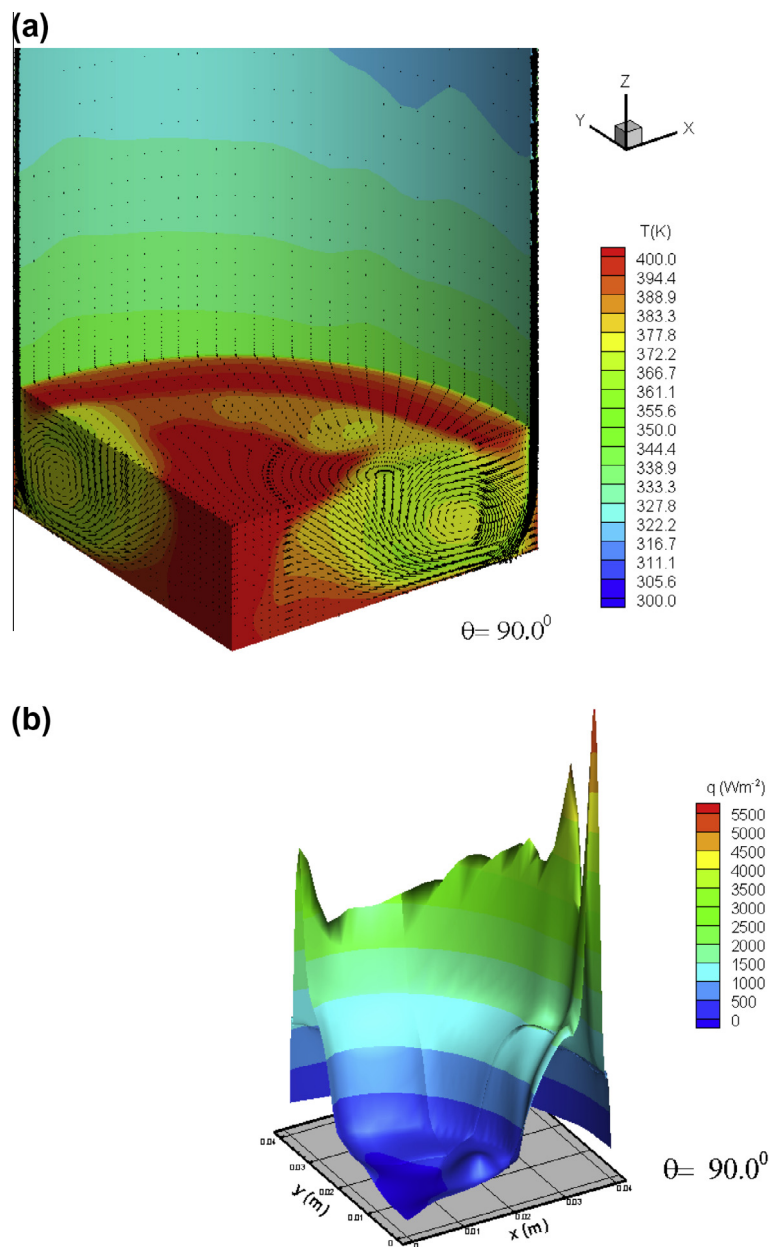


Fig. 6. Temperature contours, velocity vectors, and local heat flux distributions in the expansion chamber at $\theta = 90.0^\circ$; (a) temperature contours and velocity vectors, (b) local heat flux distributions at bottom plate.

chambers in some second-order models is apparently too crude to represent the real heat transfer behaviors inside the engine. Therefore, the attention is directed towards the investigation on the engine's heat transfer characteristics which can only be analyzed in detail by using a CFD approach.

From the expansion and compression chambers' point of view, both go through one phase of receiving gas injection from the other chamber and the other phase of gas ejection to the other chamber through the regenerative channel. In the following, these phases are termed "injection phase" and "ejection phase", respectively. In general, the injection phase of the expansion chamber is the ejection phase of the compression chamber, and vice versa. In the following, the discussion on the flow and heat transfer characteristics is divided into injection phase, ejection phase, and in regenerative channel subsections.

4.1. Injection phase

The injection phase in the individual chamber generally coincides with the expansion process of its chamber volume. In the expansion chamber, this is from $\theta = 0.0^\circ$ to 180.0° ; and in the compression chamber, it is from $\theta = 156.6^\circ$ to 347.4° . As shown in Fig. 5, this phase in the individual chamber can be characterized by the appearance of a major thermal plume in that chamber. The injection phase poses significant impact on the heat transfer in both chambers. To demonstrate this effect, Fig. 6 shows the enlarged views of velocity vectors and temperature contours as well as local

heat flux distributions in the expansion chamber at $\theta = 90.0^\circ$. Fig. 6(a) show that gas injection produces a jet of cold gas that impinges on the bottom heated wall of the chamber (the hot-end wall). This impingement brings the cold gas in direct contact with the hot-end wall, resulting in impingement heat transfer which is well known to be the most effective convective heat transfer mechanism. Therefore, the cold gas get injected into the expansion chamber is rapidly heated up. This can be verified from the distributions of the local heat flux per unit area at $\theta = 90.0^\circ$ given in Fig. 6(b). The local heat flux q is calculated by:

$$q = -k \frac{\partial T}{\partial n} \quad (17)$$

In this equation, the heat transfer from solid wall (surrounding) to gas (thermodynamic system) is positive, and that from gas to wall is negative. With referencing to the velocity vectors in Fig. 6(a), pronounced elevation of local heat flux can be seen in Fig. 6(b) at the location where the cold gas impinges on the hot bottom wall. The maximum local heat flux on the hot-end wall can be as high as 5.4 kWm^{-2} . Fig. 6(b) also shows that large local heat flux only occurs at the near vicinity of the impingement region, and the magnitude of local heat flux quickly decreases towards the center of the bottom plate. The plume of cold gas forms a primary vortex which in turn induces some very small secondary vortices at its sides. These secondary vortices promote mixing of cold and hot gas and further facilitate convective heat transfer in the expansion chamber. However, their effects are much

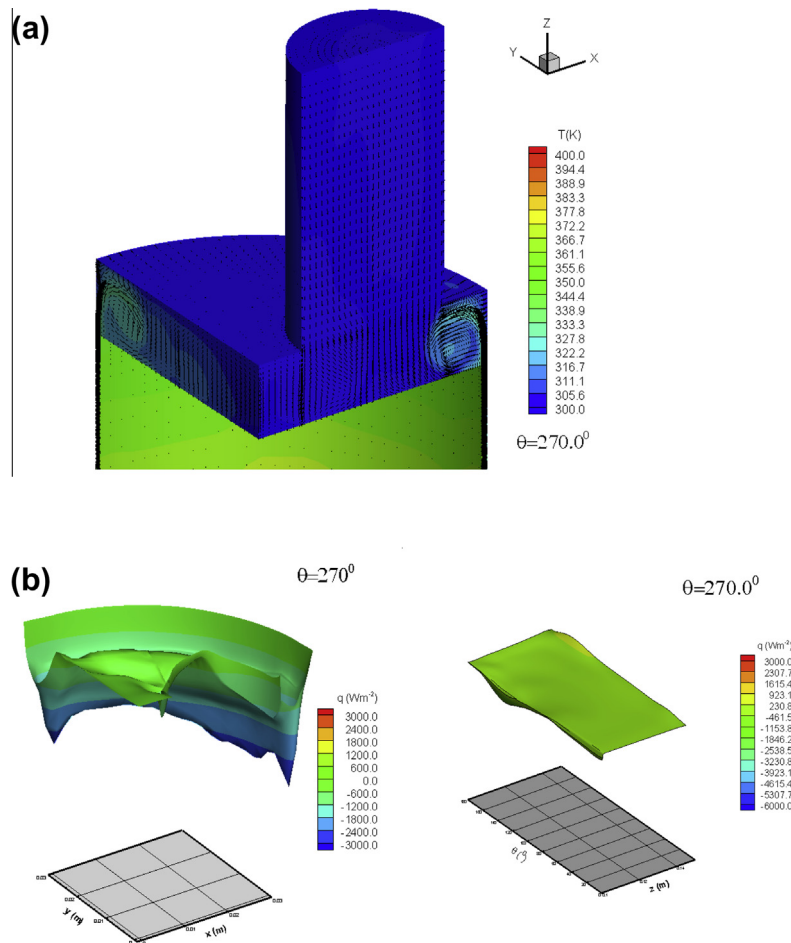


Fig. 7. Temperature contours, velocity vectors, and local heat flux distributions in the compression chamber at $\theta = 270.0^\circ$; (a) temperature contours and velocity vectors, (b) local heat flux distributions at top plate and power cylinder wall.

smaller compared to that of the impingement. The local heat flux distributions indicate that the dominant heat transfer mechanism in the injection phase of the expansion chamber is the impingement of the cold gas on the hot-end wall.

Fig. 7 shows the enlarged views of velocity vectors, temperature contours, and local heat flux distributions on the top plate of the compression chamber at $\theta = 270.0^\circ$. Similar impingement phenomenon can be observed during the injection phase of the compression chamber, but the impingement results in rapid cooling (local heat flux is negative) of the hot gas that enters the compression chamber. Here the maximum absolute local heat flux at the top plate during the injection phase is 3.0 kWm^{-2} . This is much smaller than that in the expansion chamber during its injection phase. However, due to the compression effect posed by the movement of power pistons, there is large negative local heat flux occurring at the ejection phase, which will be discussed in the next sub-section. Here, the impingement heat transfer remains the dominant heat-transfer mechanism in the compression chamber during its injection phase.

4.2. Ejection phase

The ejection of gas in expansion or compression chambers is mainly caused by the reduction of volume in the individual chamber. By referring to Fig. 4, this phase is from $\theta = 180.0^\circ$ to 360.0° in the expansion chamber and from $\theta = 347.5^\circ$ to 156.5° in the compression chamber. As mention earlier, the reduction of chamber volume is due to the motions of the displacer and power pistons.

When displacer moves towards the hot end, the volume of expansion chamber is reduced. On the other hand, when displacer and power pistons move toward each other, the volume of compression chamber is reduced. In this manner, the displacer and power pistons reduce the volume in the individual chamber by decreasing the distance between the upper and lower boundaries of that chamber, and this is equivalent to push the gas against the solid walls. In the expansion chamber, the lower wall is the hot end, while in the compression chamber; the cold end includes the upper wall and the walls of power cylinders. The pushing of gas by the displacer and power pistons creates an effect similar to impingement that largely compresses the isothermal lines towards the hot or cold ends and increases the local temperature gradient. As a result, it also promotes heat transfer in a way similar to flow impingement does. It can be seen in Fig. 4 that in the compression chamber, the ejection phase mostly coincides with the compression process of the entire engine volume, and the negative heat transfer is further boosted by the compression effect which heats up the gas in the compression chamber and creates even larger temperature difference between gas and cold-end wall. The combined heat transfer effects can be demonstrated by velocity vectors, temperature contours, and local heat flux distributions in the compression chamber at $\theta = 54.0^\circ$ shown in Fig. 8. The temperature contours in Fig. 8(a) show that the core temperature in the compression chamber reaches 330.0 K due to the compression process. Consequently, the negative local heat flux on top plate and wall of power cylinder is largely boosted as can be seen in Fig. 8(b). The absolute maximum heat flux reaches 6.8 kWm^{-2} ,

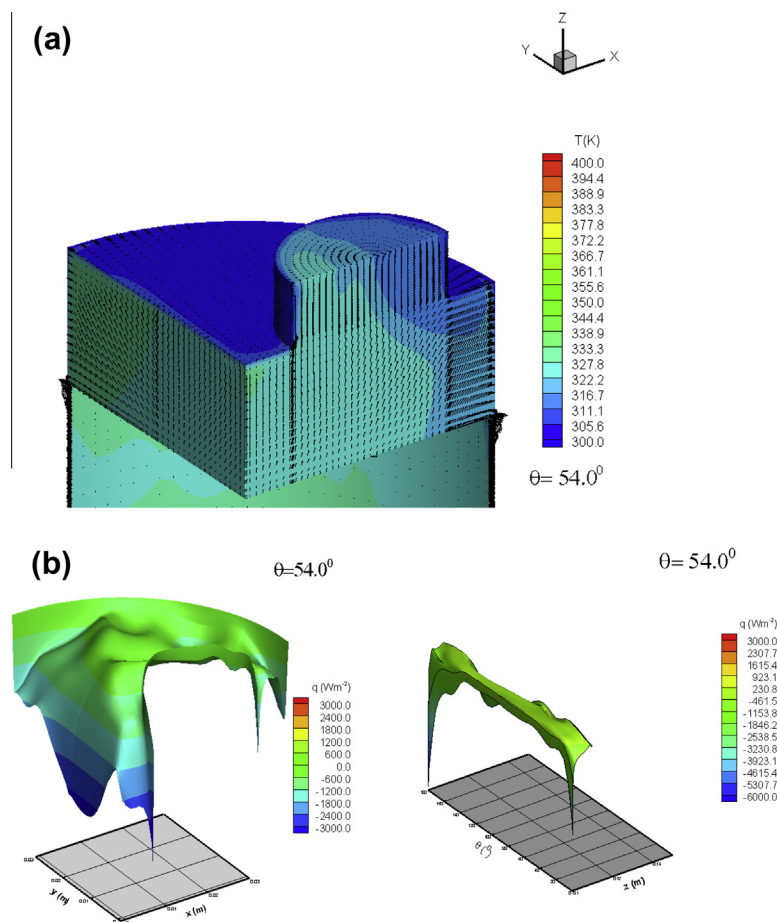


Fig. 8. Temperature contours, velocity vectors, and local heat flux distributions in the compression chamber at $\theta = 54.0^\circ$; (a) temperature contours and velocity vectors, (b) local heat flux distributions at top plate and power cylinder wall.

which is more than twice the maximum value during the injection phase. In the expansion chamber, on the other hand, the ejection phase does not coincide so well with the expansion process of the entire engine volume; therefore, the promotion on positive heat transfer by the mechanism of enlarging temperature difference between gas and hot-end wall via gas expansion is not very significant. On the contrary, Fig. 4 shows that from $\theta = 269.0^\circ$ to 360.0° , the ejection phase of the expansion chamber coincides with the compression of total engine volume, thus the gas inside the expansion chamber is heated not only by the hot-end wall but also by the compression effect. As a result, the local maximum temperature in the expansion chamber reaches as high as 417.0 K, which is higher than the hot-end temperature. The elevated gas temperature even gives rise to small negative heat flux on bottom plate. Due to the above factors, the local heat flux on the bottom plate is moderate during the ejection phase of the expansion chamber.

4.3. In regenerative channel

The major function of a regenerative channel is to pre-heat the gas during the injection phase of the expansion chamber and to pre-cool the gas during the injection phase of the compression chamber. From the local heat flux distributions on the wall of displacer cylinder shown in Fig. 9, it can be seen that the regenerative channel performs this task relatively well as the heat flux during the injection phase of expansion chamber on regenerative wall shown in Fig. 9(a) is mostly positive (heating gas), and that during the injection phase of compression chamber shown in Fig. 9(b) is mostly negative (cooling gas). However, there are exceptions. In terms of the injection phase of the expansion chamber, Fig. 9(a) shows that negative heat flux is present at the upper section of the displacer cylinder wall. This is because gas at a higher temperature than the wall temperature entering the regenerative channel. The reason for the elevated gas temperature is due to the heating from the compression of the entire engine volume. In terms of the ejection phase of the compression chamber, Fig. 9(b) shows that at the lower section of displacer cylinder wall (near entrance of the regenerative channel), heat flux is positive. This happens when the gas with a temperature lower than the wall temperature in the expansion chamber entering the regenerative channel. The lower gas temperature in the expansion chamber is due to the cooling of gas by the expansion of the entire engine volume. Due to the eccentric location of the power cylinders, Fig. 9 again shows that the local heat flux distribution is not axisymmetric, especially near both entrances of the regenerative channel.

4.4. Average and integrated quantities

The p_{ave} – V diagrams of the expansion and compression chambers are illustrated in Fig. 10. Here p_{ave} is the averaged pressure in the individual chamber. It can be seen that during one engine cycle, the expansion chamber yields positive work, whereas the compression chamber produces negative work. The absolute value of the positive work is larger than that of the negative work, giving rise to positive net work output to the engine's shaft. The variations of average temperature in the expansion and compression chambers are shown in Fig. 11(a), while those of heat flux are shown in Fig. 11(b). Here heat transfer rate Q is calculated by integrating the local heat flux q over the individual chamber's wall surface. Fig. 11(a) indicates that average temperature fluctuation in the expansion chamber is within the range of 28 K, and due to the compression effect mentioned earlier, the maximum average temperature is 401.0 K which is slightly higher than the hot-end temperature. In the compression chamber, the average temperature fluctuation is within the range of 20 K. Both average temperature

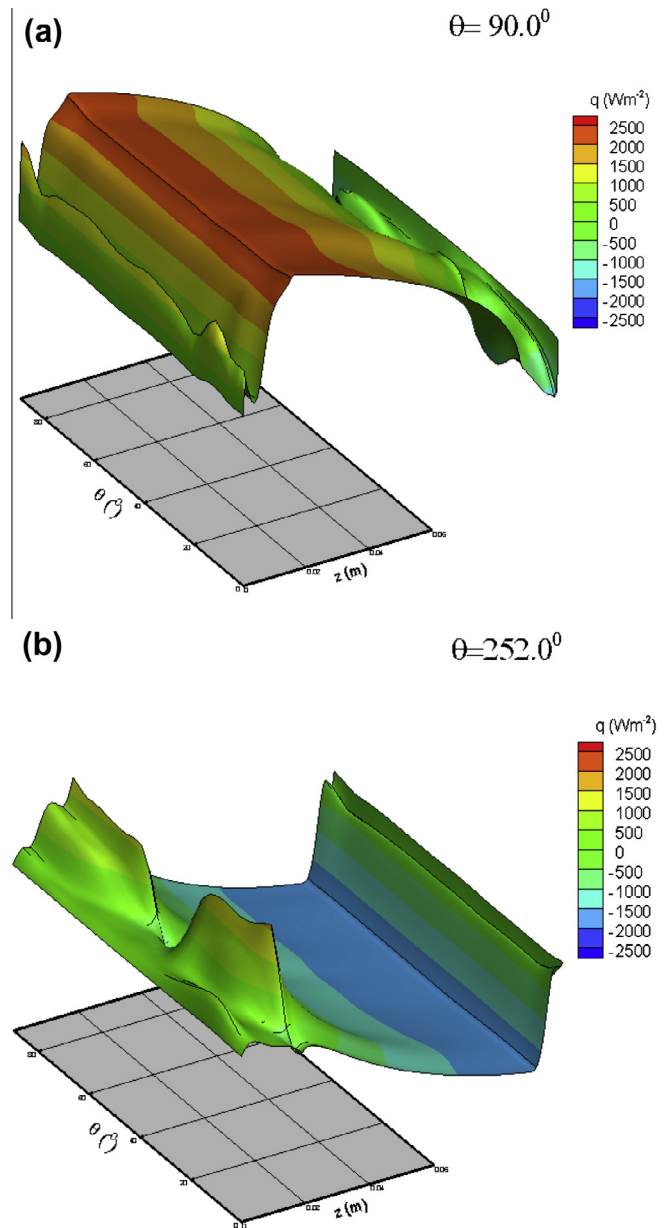


Fig. 9. Local heat flux distributions on the regenerative wall at two different crank angles; (a) $\theta = 90.0^\circ$, (b) $\theta = 252.0^\circ$.

variations show only one peak and one valley, corresponding to one injection phase and one ejection phase within an engine cycle. In Fig. 11(b), it can be seen that heat transfer rate in the expansion chamber in the injection phase first increases steadily, reaches the maximum, and then decreases monotonously. In the ejection phase, however, the general tendency is decreasing but it briefly rebounds twice during the descent. This is due to the fact that the second half of the ejection phase of the expansion chamber generally coincides with the expansion of the entire engine volume, and the expansion effect cools down the gas temperature, thus promoting positive heat transfer. The amount of heat transfer rate (in terms of absolute value) in the compression chamber also shows monotonous increasing then decreasing in its injection phase. In its ejection phase, the general tendency is decreasing but there it also briefly rebounds once due to a period of coincidence of the ejection phase with the compression of the entire engine volume as described earlier in the Section 4.3.

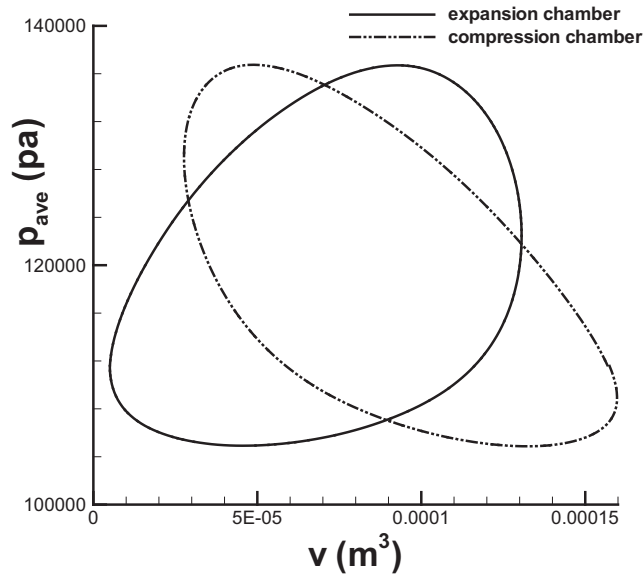


Fig. 10. p_{ave} - V diagram in the expansion and compression chambers.

Due to the expansion and compression of engine volume, heat input and output can happen at the hot end, cold end, and on the wall of regenerative channel. Therefore, the integration of heat input cannot be confined just along the wall of the expansion chamber. Neither can the integration of heat output be confined along the wall of compression chamber. The rates of heat input and output are calculated by integration along all solid boundaries:

$$Q_{in} = \frac{\omega}{2\pi} \int_{t=t_0}^{t=t_0+t_p} \int_{wall} q dA dt, \quad \text{if } q > 0, \quad (18)$$

$$Q_{out} = \frac{\omega}{2\pi} \int_{t=t_0}^{t=t_0+t_p} \int_{wall} q dA dt, \quad \text{if } q < 0. \quad (19)$$

In terms of the evaluation of output power, the focus is on the surface of the power piston because this is the exact place engine work is delivered. The power is calculated by:

$$W = -\frac{\omega}{2\pi} \oint \left(\int_{piston} p dA \right) dz, \quad (20)$$

where the negative sign on the right hand side of equation is to account for the negative ω in this study, and the circular integration is from minimum z_p to maximum z_p and back to minimum z_p . The calculated rates of heat input, heat output, and power are 13.28 W, 12.63 W, and 0.64 W, respectively, resulting in a thermal efficiency η of 4.82%. Here, η is calculated by dividing the engine power by the rate of heat input. Such a low efficiency is mainly due to the small temperature difference between the hot and cold ends and the low engine speed. Since the focus of the study is to understand the heat transfer characteristics of the Stirling engine cycle, to improve the engine's efficiency will be dealt with in some future study.

To help readers understand the dynamic processes inside the current Stirling engine, two videos have been made. The video named "gamma_all", plays the transient variations of temperature contours and velocity vectors in expansion and compression chambers within two engine cycles. The other video named "flux_all" plays the variations of local flux on bottom plate, top plate, power cylinder wall, and regenerative wall, within two engine cycles.

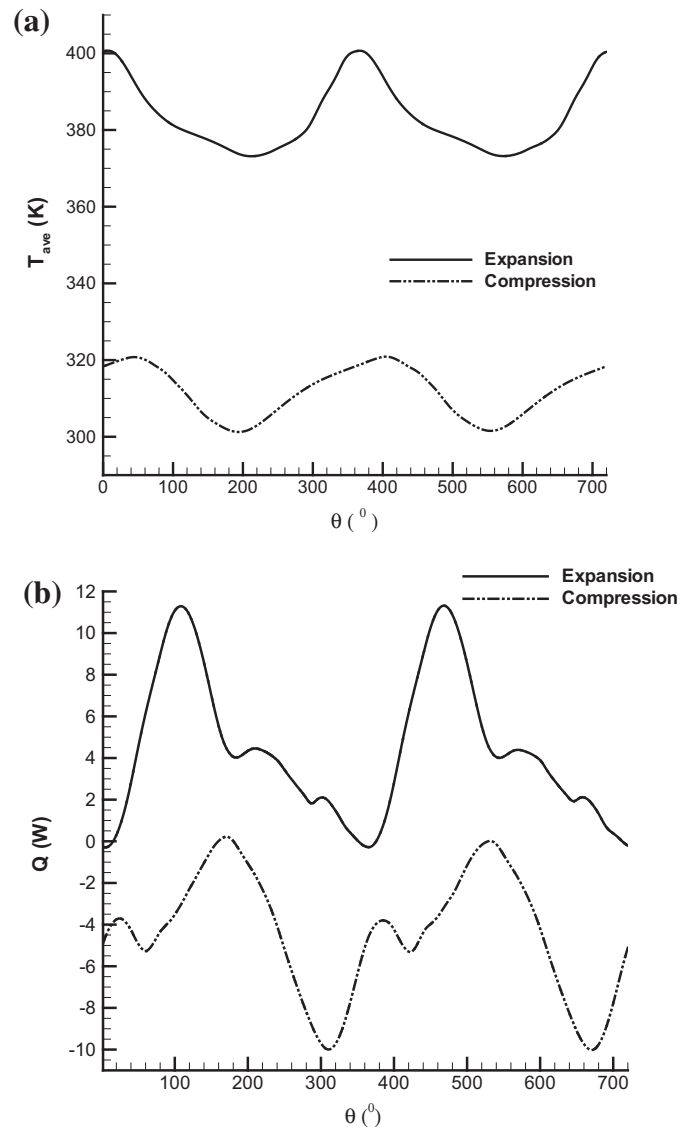


Fig. 11. Variations of average temperatures and heat transfer rates in the expansion and compression chambers; (a) variations of average temperatures, (b) variations of heat transfer rates.

5. Conclusion remarks

An in-house CFD code has been developed and applied to a twin-power-piston γ -type Stirling engine to study the heat transfer characteristics in the thermodynamic cycle of the Stirling engine. This is the first CFD simulation on this engine, and the heat transfer characteristics have been investigated in great detail. The results include temperature contours, velocity vectors, and distributions of local heat flux at several important stages in an engine cycle as well as average temperature variations, integrated rates of heat input, heat output, and engine power. Some conclusion remarks from this study can be drawn as follows:

1. The complicated heat transfer behaviors in the engine result in highly non-uniform temperature distributions in all sections of the engine at most of the time in an engine cycle. The assumption of uniform chamber temperatures and the use of constant heat transfer coefficients to account for heat transfer adopted by some zero- or one-dimensional models are too crude to reflect the reality.

2. In the injection phase, impingement heat transfer is the major heat transfer mechanism which allows gas to be heated (in expansion chamber) or be cooled (in the compression chamber) very effectively. However, this effect tends to be concentrated at some localized region, resulting in non-uniform heating or cooling of gas.
3. In the ejection phase, the motions of displacer and power piston also create a sort of heat transfer effect similar to impingement, but this effect acts on a much larger region and also promotes heat transfer effectively especially in the compression chamber where the negative heat transfer is further boosted by the rising of gas temperature due to compression of the entire engine volume.
4. The regenerative channel functions as it was designed for at most of the time within an engine cycle. It pre-heats or pre-cools gas before gas got injected into the expansion or compression chamber. However, exceptions can occur near both regenerative channel entrance regions at some brief moments.

Conflict of interest

We hereby declare that conflict of interest does not exist regarding the research work documented in our manuscript titled “A computational fluid dynamics study on the heat transfer characteristics of the working cycle of a low-temperature-differential γ -type Stirling engine”.

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Appendix A. Supplementary data

Supplementary data associated with this article can be found, in the online version, at <http://dx.doi.org/10.1016/j.ijheatmasstransfer.2014.03.055>.

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